

## STUDY OF GRIDDING AND CELL-CELL INTERACTIONS IN THE METHOD OF MOMENTS ANALYSIS OF ARBITRARILY SHAPED PLANAR CIRCUITS

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### Abstract

A method of Galerkin analysis of polygonal planar circuits embedded in a multilayered medium is presented. The circuit is gridded up using rectangular and triangular cells. Rooftop basis functions are used. The paper focuses on an efficient calculation scheme for the cell-cell interactions. A microstrip low-pass filter is investigated and comparisons are made with measured and other simulated data. Grid accuracy and convergence are discussed.

### 1. Introduction

The method of moments (MoM) has been widely used for the analysis of planar microwave circuits. Several MoM software packages for the analysis of planar circuits are commercially available [1]-[4]. Most of them have the restriction that a uniform rectangular background grid is used. This often results in an approximation of the geometry of the physical structure. Only recently, research efforts have been focused on mixed grids of rectangles and triangles [5].

In this paper a general technique is presented which is able to simulate the circuit parameters of arbitrarily shaped microstrip circuits in a multilayered medium. The method of Galerkin is applied for the solution of the mixed potential integral equation governing the behaviour of the circuit. The grid consists of rectangular and triangular cells. An efficient calculation technique for the cell-cell interactions will be presented in section 3 after the formulation of the integral equation in section 2. In the last section a low-pass filter is simulated and different grids are used which fit into the physical structure. Simulated results are compared with measured data and simulated data obtained with the circuit-simulator MDS [6].

### 2. Theory

The algorithm is based on the application of the method of moments to a mixed-potential integral equation (1) for the current distribution on the microstrip structure S :

$$\int_S dS' [G^A(\mathbf{r}, \mathbf{r}') \mathbf{J}(\mathbf{r}') - \nabla(G^V(\mathbf{r}, \mathbf{r}') \nabla' \cdot \mathbf{J}(\mathbf{r}'))] = -\mathbf{E}^{in}(\mathbf{r}) \quad (1)$$

where  $\mathbf{E}^{in}$  is the incoming field. The integral kernels are the electric  $G^V$  and magnetic  $G^A$  Sommerfeld potentials of a point source in the layered medium. A mixed grid of rectangular and triangular cells is used to subdivide the planar structure. Rooftop basisfunctions  $\mathbf{b}_i(\mathbf{r})$ ,  $i=1,\dots,M$  [5] are used to model the current. The system matrix equation  $\mathbf{Z} \cdot \mathbf{I} = \mathbf{V}$  follows from the testing of the integral equation with rooftop functions (Galerkin testing). The solution of the matrix equation yields the current from which the circuit parameters can be de-embedded.

### 3. Cell-cell coupling elements.

The elements of the system matrix  $\mathbf{Z}$  determine the coupling between an excitation and an observation cell. These cell-cell impedances are of the form :

$$Z_{ij} = \int_{S_i} dS \int_{S_j} dS' [G^A(\mathbf{r}, \mathbf{r}') \mathbf{b}_i(\mathbf{r}) \cdot \mathbf{b}_j(\mathbf{r}') + G^V(\mathbf{r}, \mathbf{r}') \nabla \cdot \mathbf{b}_i(\mathbf{r}) \nabla \cdot \mathbf{b}_j(\mathbf{r}')] \quad (2)$$

where  $\mathbf{r}$  and  $\mathbf{r}'$  are position vectors in  $S_i$  and  $S_j$ . In order to calculate  $Z_{ij}$  the potentials  $G^A$  and  $G^V$  in (2) are curve-fitted into a power series of  $p=|\mathbf{r}-\mathbf{r}'|$ . For the self-patch and nearby coupling elements, the  $1/p$  singularity in the Green's potentials is taken into account explicitly. The number of power terms depends on the lateral separation of the excitation and the observation cell and the error that is allowed for the cell-cell coupling. These two dependencies

are translated into a circle of influence for each power term.

Since the rooftop functions  $\mathbf{b}_i(\mathbf{r})$  are linear functions in  $(x, y)$ , the basic double surface integrals involved in (2) are of the form:

$$Q(i, j, p) = \int_{S_i} dS \begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} \int_{S_j} dS' \begin{Bmatrix} 1 \\ x' \\ y' \end{Bmatrix} p^p \quad (3)$$

An analytical scheme has been implemented for the quadruple moment integrals (3). In the analytical treatment a discrimination is made between close and distant coupling.

For the close coupling, i.e. selfpatch and neighbour coupling, new recurrence relations make it possible to derive the different power terms in  $p$  from the two lowest ones, i.e. higher order terms are calculated at very low computational cost. This approach combined with the circles of influence is more efficient than the technique described in [5].

For distant coupling, i.e. typically for two cells more than 10 cells apart, the power term  $p^p$  is expanded in a Taylor series around the distance between the centre of gravities of the two cells. Very simple and computational efficient single integrals emerge. Distant cell-cell coupling terms (two terms in the Taylor series) are typically calculated a factor 50 faster than selfpatch coupling terms. This new scheme for the distant cell-cell coupling makes

our technique very well-suited for the simulation of large circuits.

#### 4. Results

In this section results are presented for a microstrip low-pass filter (Figure 1) printed on a 25 mil alumina substrate with dielectric constant  $\epsilon_r = 9.6$ . The filter is simulated using the presented technique with the three different grids shown in Figure 1(a), 1(b) and 1(c). The grids differ from each other in the number of cells across the width of the lines. The longitudinal cell sizes are smaller than  $\lambda/20$ . In the asymmetric cross junctions triangles are used. Figure 2 displays measured and simulated  $|S_{21}|$  as a function of frequency. The figure clearly shows the convergence of the simulations. The results for the coarsest grid (mixed grid 1) are already very accurate. Table 1 shows the number of unknowns and the computation time for a simulation with the three grids. The total CPU time needed for the calculation of  $N$  frequency points equals the amount given in the second column plus  $N$  times the value in the third column.

In Figure 3 comparison is made between simulations made with the mixed grid of Figure 1(c) and the uniform grid of Figure 1(d) and with an MDS circuit simulation [6]. The uniform grid requires an approximation of the physical structure. This approximation of the geometry

slightly shifts the results as can be seen in Figure 3. The use of a uniform background grid also increases the number of unknowns resulting in a higher CPU time/frequency point (see Table 1). This example stresses the importance of a flexible grid. The results from MDS also deviate from the electromagnetic simulation of the structure in Figure 1(c). This is primarily due to the absence of parasitic field coupling between the different components of the filter in a circuit simulator.

#### 5. References

- [1] em, Sonnet Software, Liverpool, NY
- [2] SFPMIC+, Jansen Microwave, Ratingen, Germany
- [3] EmSim, EeSof, Westlake Village, CA
- [4] Explorer, Compact Software, Paterson, NJ
- [5] J. X. Zheng, "Electromagnetic Modeling of Passive Circuit Elements in MMIC," University of Colorado at Boulder, PhD dissertation, January 1990.
- [6] MDS, Hewlett-Packard, Palo Alto, CA

	number of unknowns	CPU time (freq. independent)	CPU time / frequency point
mixed grid 1	80	1m46s	1m23s
mixed grid 2	193	4m17s	2m04s
mixed grid 3	360	5m15s	2m37s
uniform grid	604	2m22s	4m51s

Table 1. Comparison of the CPU times (HP-series 700 workstation) for the different grids shown in Figure 1.

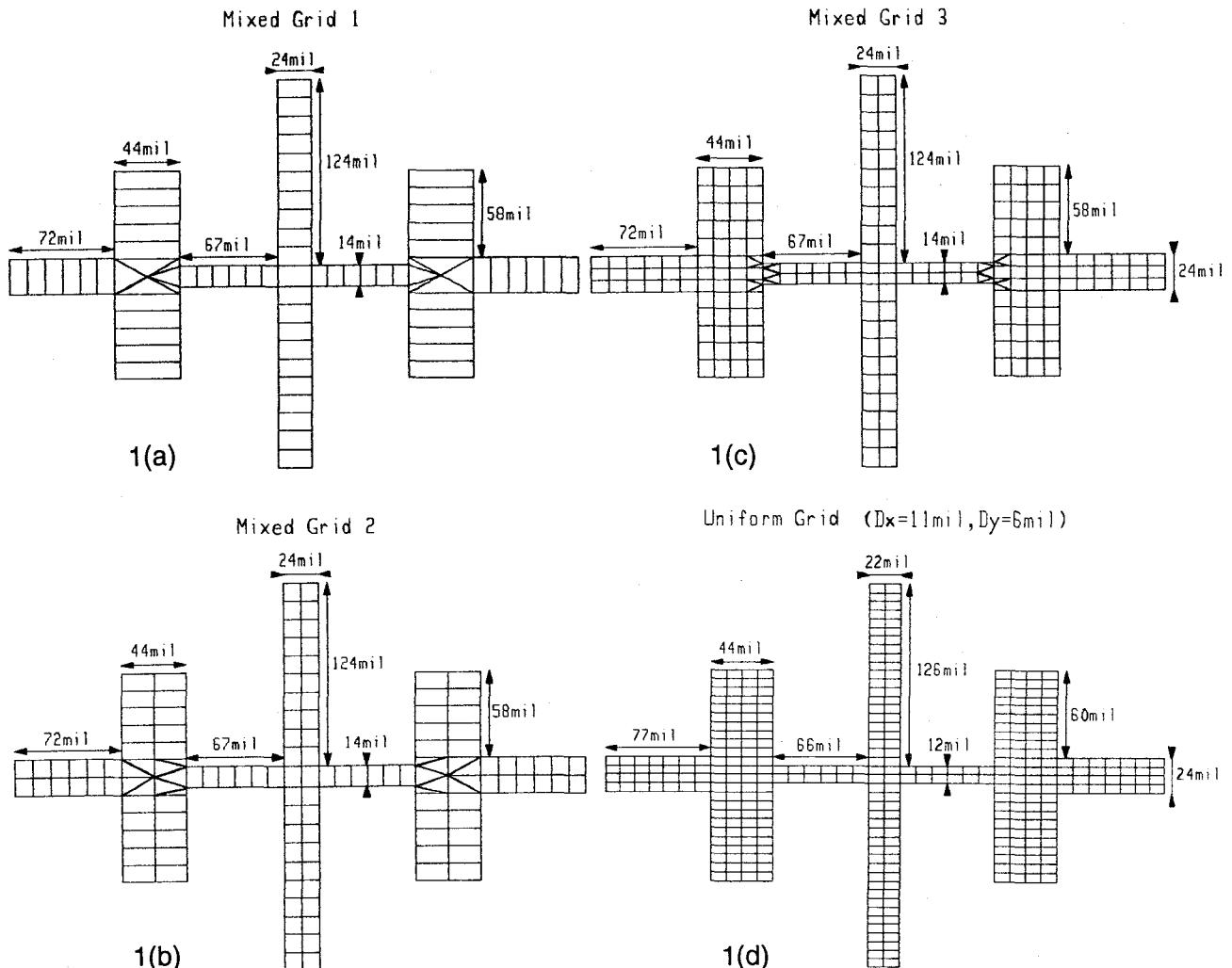


Figure 1. Plot of the different grids used to simulate the low-pass filter.

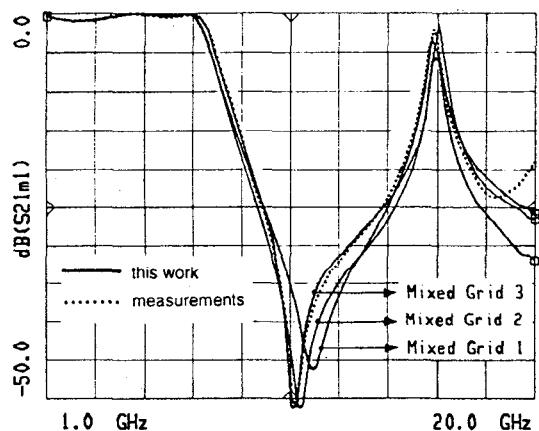


Figure 2. Magnitude of the S<sub>21</sub> scattering parameter calculated with the mixed grids 1, 2 and 3 and comparison with measurements.

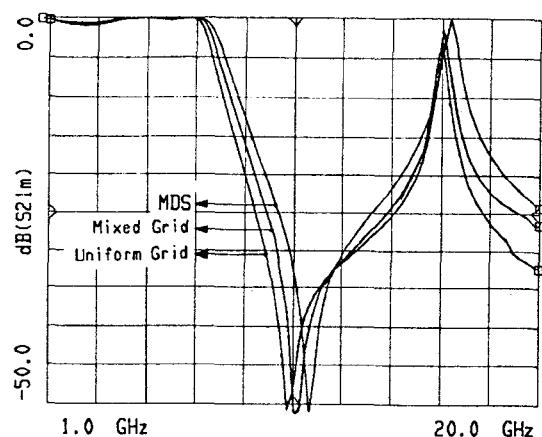


Figure 3. Magnitude of the S<sub>21</sub> scattering parameter calculated with a mixed grid, an uniform grid and comparison with circuit simulation (MDS)